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EVALUATION OF REINFORCEMENT NEEDS IN SHALLOW FOUNDATIONS, COMPARING MODULUS OF SUBGRADE REACTIONS AND FEA

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Abstract

This paper presents a study on how the use of subgrade reactions can have an effect on the design of shallow foundations. A numerical example that compares the use of subgrade reactions with full finite element analysis is presented. The paper demonstrates how the advanced three-layer model, first introduced to compute the reinforcement needs in concrete plates and shells can be applied to concrete foundations. After a brief introduction to the concept of subgrade reactions, the paper concludes with a numerical study demonstrating the application of the advanced three-layer model. The paper is ended with a discussion of potential hazards of using subgrade reactions to compute reinforcement needs.

Keywords: Modulus of subgrade reactions, soil-structure interaction, finite element analysis

1 Introduction

In the last decades, numerical methods, i.e. FEA have become a common tool for both structural and geotechnical engineers. To create a realistic model of any structure it is essential to include the interaction between the ground and the structure, as it can have a large influence on the overall response [1]. As the use of computational methods has increased, the models to analyse soil behaviour and structural response have got increasingly sophisticated. When designing foundations, it is however a common practice, that separate numerical models are used to analyse the soil behaviour and the structural response.

A common procedure is that the geotechnical engineer establishes a model of the site conditions and performs a simulation of the behaviour of the ground using pre calculated load values received from the structural engineer. The resulting settlements can then in turn be used in the dimensioning of the structure. Using separate models can lead to unrealistic prediction of the behaviour of both structure and load, as the soil-structure interaction is disregarded.

During the design of the structural elements, a common method to model the soil-structure interaction is to idealise the soil as a series of independent springs. The spring stiffness is often referred to as the modulus of subgrade reaction and was first introduced by Winkler [2]. However, the Winkler model has some well-documented drawbacks, e.g. for a slab with a uniform load the Winkler model gives a uniform displacement, resulting in a no bending moments or shear forces [3]. Another issue with the Winkler model is that it is difficult to determine a suitable value of the modulus of subgrade reaction to represent the soil behaviour. Over the years a number of researchers have suggested modifications to the Winkler model, i.e. Pasternak [4] and Vlasov and Leont'ev [5].

Even though the Winkler model has some well-known shortcomings, it continues to be a common method for practicing engineers. It is therefore, important to enlighten how these simplifications affect the design of common structures such as foundations. In this work we have evaluated how using subgrade reactions during the design of shallow foundations can effect the dimensioning of the reinforcements. Numerical studies using the modulus of subgrade reaction to idealise the behaviour of the underlying soil have been compared to full finite element analysis (FEA). To evaluate the reinforcement requirements in the foundations, the advanced three-layer model, (ATLM), described in [6, 7], has been implemented and used within a general purpose FE-tool. The advanced three-layer model was introduced to calculate the reinforcement needs in concrete plates and shells. The model uses the bending moments m_{xx}, m_{yy} and the twisting moment m_{xy} with the in-plane normal forces n_x, n_y and shear force n_{xy} to determine whether the resisting concrete moments, m_c and forces n_c in the top- and bottom-layers are exceeded, and thereby reinforcement is needed. For a more extensive explanation the reader is referred to [6].

2 Modulus of subgrade reaction

The Winkler model is still widely used by practicing engineers to model the interaction between soil and structures. The idea behind the model is to remove the subgrade from the structural analysis, replacing the soil with a series of springs. The spring stiffness is often referred to as modulus of subgrade reaction, k_s and was first introduced by Winkler [2]. The original application was to compute the stresses and deformations in railroad structures. The stiffness of the spring k_s is defined as the ratio between the vertical contact pressure p and the corresponding settlement δ, i.e.,

$$k_s = \frac{p}{\delta} \tag{1}$$

The method does however only give information on the structure, and gives no information on displacements or stress levels in the soil. Moreover, it can be difficult to determine the correct stiffness of the spring, and a number of papers have been written on the topic, e.g. [8–10]. If not fully understood, any use of idealisations and simplifications can lead to incorrect decisions. This also applies to the use of the concept of subgrade reactions during the design of foundations, which can lead to misconceptions of the structural response. Figure 1a), illustrates the response of a raft foundation subjected to a uniform load, where the use of a constant value of the modulus of subgrade reaction leads to an unrealistic response. The result from the Winkler model produces uniform displacements, without any differential settlements in comparison to a more realistic response illustrated in Figure 1b).

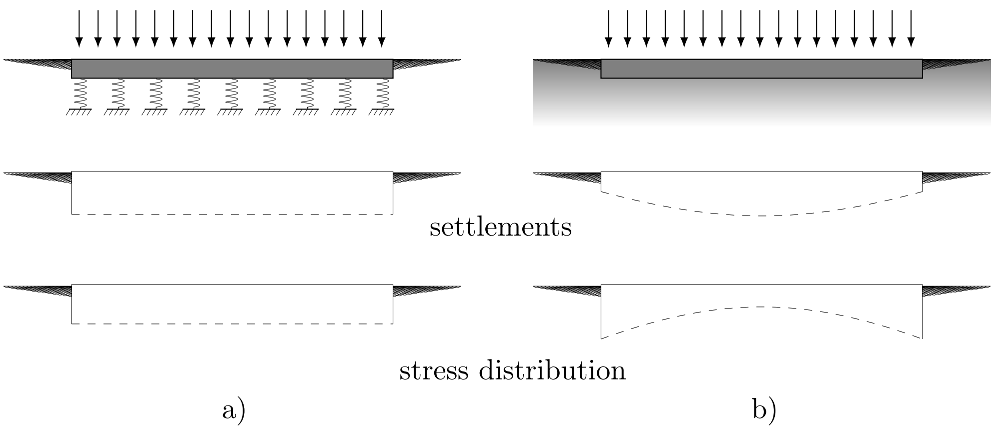


Figure 1: Illustration of predicted values, a) using springs to represent soil behaviour compared to, b) using a joint model that includes both the soil and structure.

One way of limiting the drawbacks of the Winkler model, is to let the values of k_s vary along the foundation. In this work we compare the results from a standard Winkler model with a constant modulus of subgrade reaction to the results from a modified Winkler model where we let the value of k_s vary along the foundation. The modulus of subgrade reaction for the modified version is calculated by evaluating the vertical displacements and reaction forces from a geotechnical FE analysis.

3 Numerical studies

To investigate the effects of using the Winkler model and the modified Winkler model two numerical simulation of a shallow raft foundations have been conducted. Both studies consist of a flexible raft foundation subjected to a uniform spread load according to Figure 2b. The concrete slab is 10×15 m, resting on top of a 35 m layer of clay till. The first example investigates the effect of the rigidity of the concrete slab through a parameter study of the slab thickness. In the second example, the effects of a non-uniform load is investigated, through a parameter study of the influence of the load q_1. Moreover, the Mohr-Coulomb criterion has been used to model the soil behaviour, and the material properties used to model the clay till are presented in Table 1. The material properties of the concrete slab are presented in Table 2. Furthermore, the spread load, q_2=20 kN/m^2 for all the simulations.

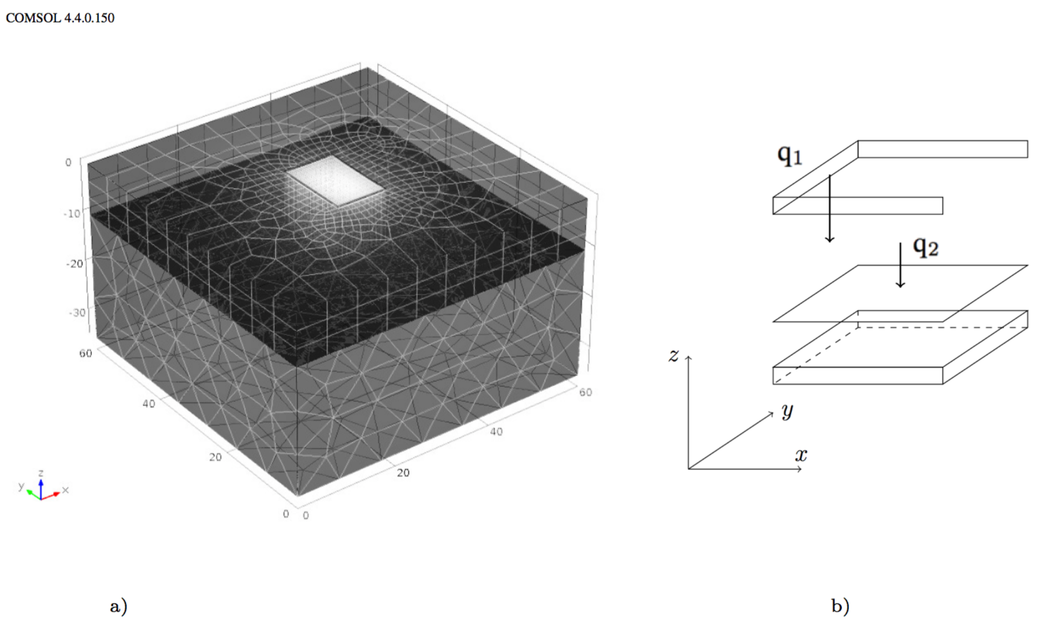


Figure 2: a) The finite element model used to determine the deformations of the soil due to the loading of the concrete slab. b) Loading condition for the concrete slab.

Table 1: Material properties for soil used in FE-model.

Material parameters		unit
Young's modulus, E	40	MPa
Posson's ratio, ν	0.25	-
Density, ρ	1900	kg/m^3
Cohesion, c	20	kPa
Angle of internal friction, ϕ	30	deg
Dilatation angle, ψ	5	deg

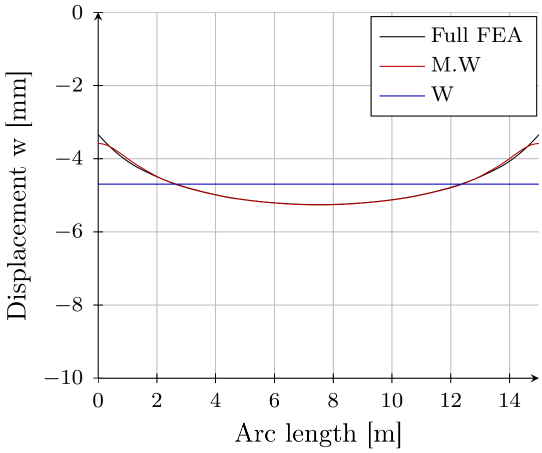
Table 2: Material properties for concrete used in FE-model.

Material parameters		unit
Young's modulus, E	25	GPa
Posson's ratio, ν	0.33	-
Density, ρ	2300	kg/m^3

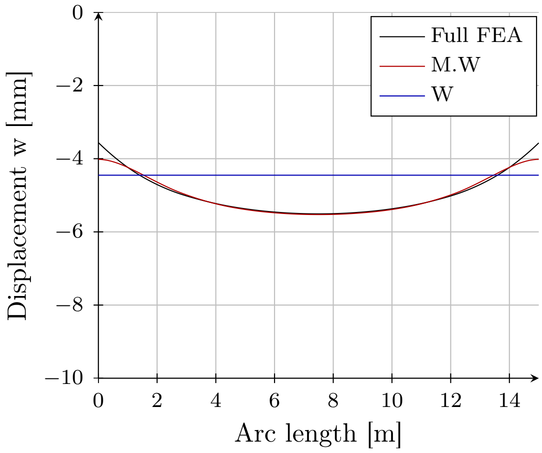
To compare how the modulus of subgrade reaction affects the dimensioning of the reinforcement in the slab a full three dimensional analysis has been carried out. In the finite element analysis, the soil-structure interaction is modelled by assuming full continuity between the surface of the concrete slab and the soil. Two different models using the subgrade modulus have been established to investigate the effect of using a constant value of k_s compared to using a value that varies along the bottom of the foundation. In the first model, the value of $k_s = \sigma_{zz}/\Delta w$ varies with the contact stress and deflections of the slab. In the second model the value of the subgrade modulus is taken as the average value of the ratio $\sigma_{zz}/\Delta w$ along the bottom of the concrete slab.

3.1 Parameter study of the effects of the slab rigidity

In the presented study the thickness of the concrete slab was increased from 50 mm to 300 mm to investigate the effects of the rigidity of the slab on the displacement field of the raft foundation, and in turn the dimensioning moments. Figure 3a) and b) show the displacement along the y-axis at centre of the slab for the 50 and 100 mm thick slabs. The results from the Winkler method using a constant value of the modulus of subgrade reaction can as expected not capture the differential settlements. On the other hand the modified Winkler model shows results in good agreement with the full finite element analysis. Moreover, through study of Figure 4 and 5 one can see that the modified model diverges from the full FE analysis more the stiffer the slab gets. It is also worth noting that, even though the modified model produces results that are in general are in good agreement with the full FE analysis, the stiffness along the edges is slightly underestimated.

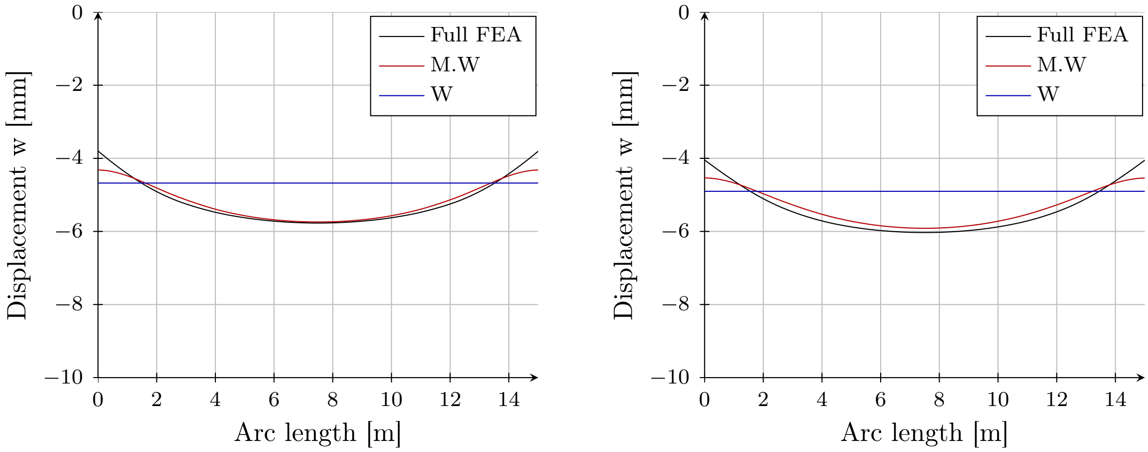


(a)



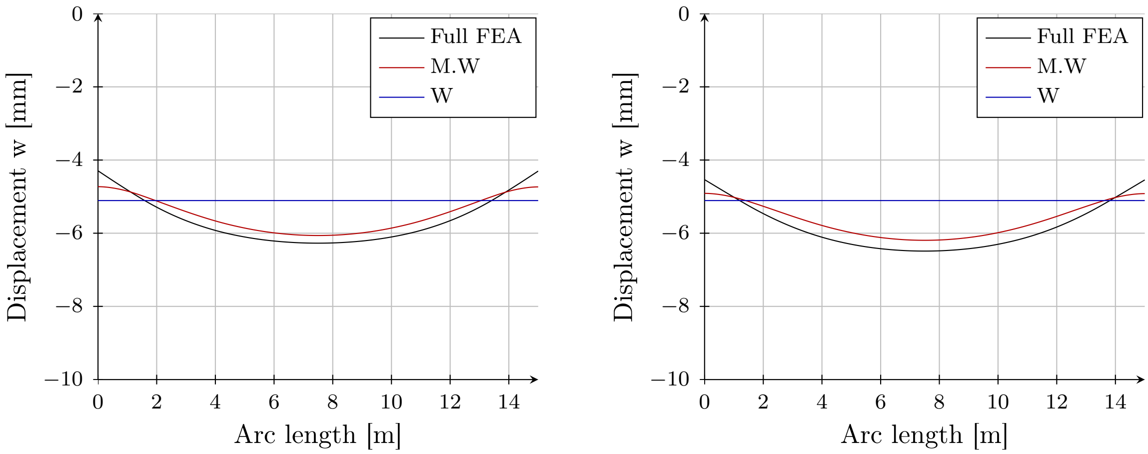
(b)

Figure 3: Vertical displacement along the centre of the x-axis of the concrete slab. a) For a concrete slab with a thickness of 50 mm. b) For a concrete slab with a thickness of 100 mm.



(a) **(b)**

Figure 4: Vertical displacement along the centre of the x-axis of the concrete slab. a) For a concrete slab with a thickness of 150 mm. b) For a concrete slab with a thickness of 200 mm.



(a) **(b)**

Figure 5: Vertical displacement along the centre of the x-axis of the concrete slab. a) For a concrete slab with a thickness of 250 mm. b) For a concrete slab with a thickness of 300 mm.

3.2 Parameter study of the effects of a non-uniform load

In the second study the thickness of the concrete slab is kept constant at 250 mm, whereas the load q_1 is increased from 0 to 50 kN/m with steps of 10 kN/m. Figure 6-8 show the displacement along the y-axis at centre of the slab. From the results presented in the figures it can be seen that the Winkler model with a constant value of the subgrade reaction cannot capture the displacement profile of the full finite element model. Moreover, the Winkler model produces an opposite displacement profile, which is more pronounced the greater the boundary load becomes. This effect of the Winkler model has previously been reported by Horvath [10], among others.

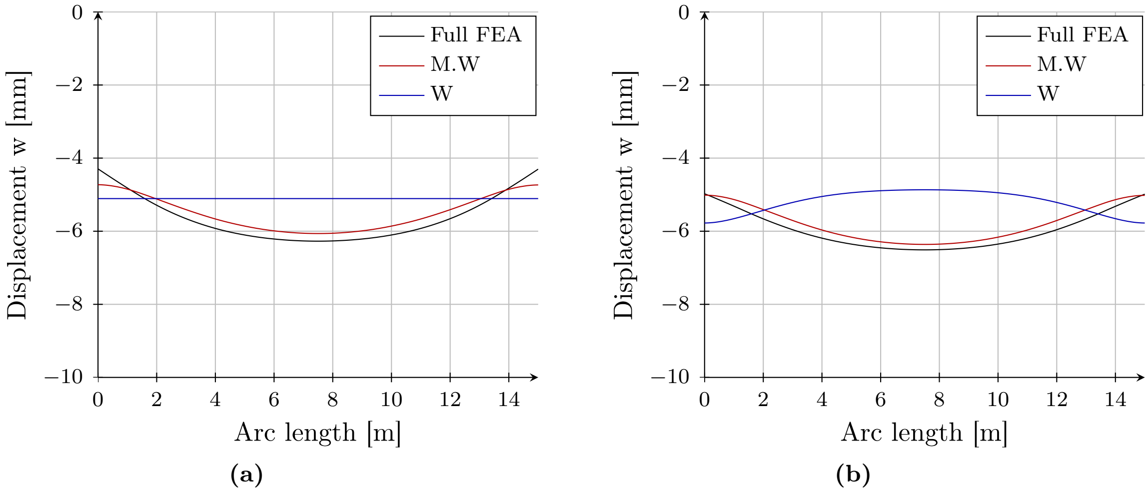


Figure 6: Vertical displacement along the centre of the x-axis of the concrete slab. a) Without boundary load. b) With a boundary load q_1=10 kN/m.

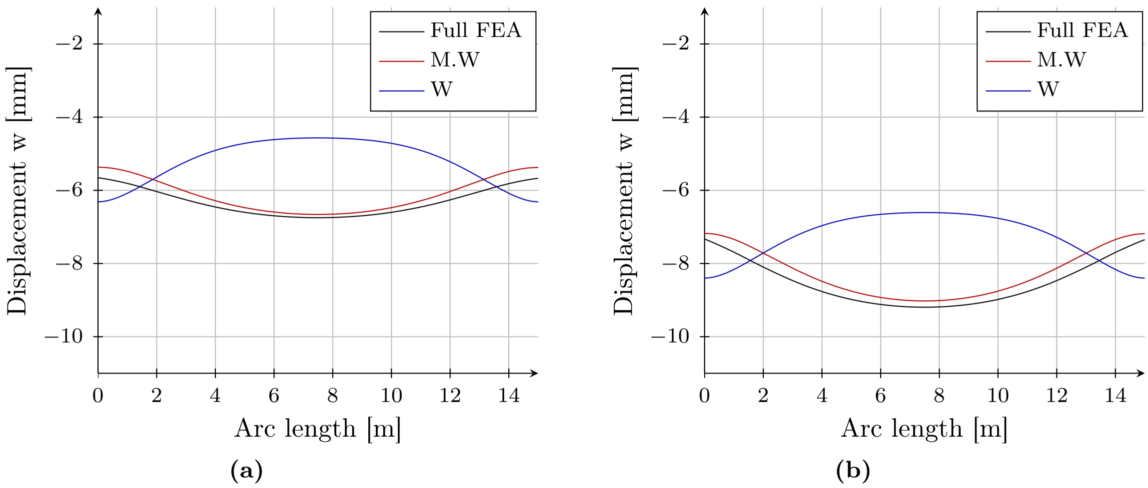


Figure 7: Vertical displacement along the centre of the x-axis of the concrete slab. a) For load case, $q_1 = 20$ kN/m. b) For load case, $q_1 = 30$ kN/m.

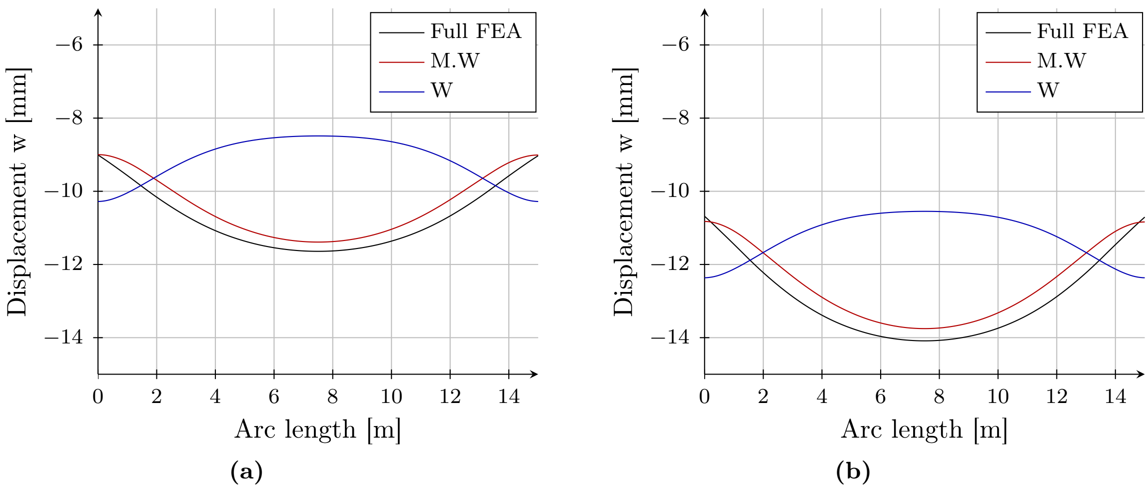


Figure 8: Vertical displacement along the centre of the x-axis of the concrete slab. a) For load case, $q_1 = 40$ kN/m. b) For load case, $q_1 = 50$ kN/m.

Even though the resulting displacements from the modified Winkler model are in good agreement with results from the full FE analysis, a closer look at the full displacement field gives a different view. Figure 9 shows the displacement from the three models. From the displacements it becomes evident that the model using a uniform modulus is in poor agreement, both in magnitude and shape of the displacements. The difference between displacements of the full finite element analysis and the model that a variable value of the subgrade modulus is not as large, but still visible.

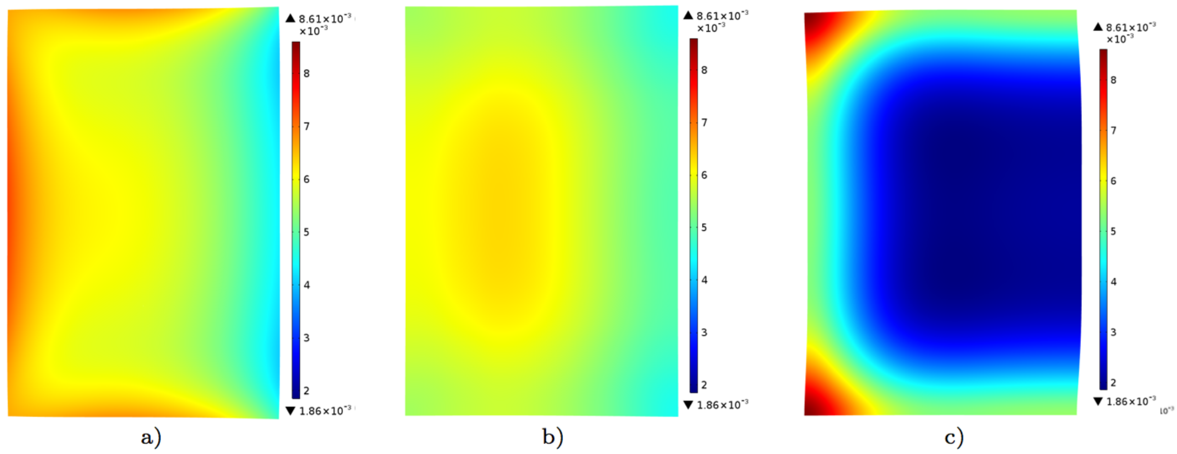
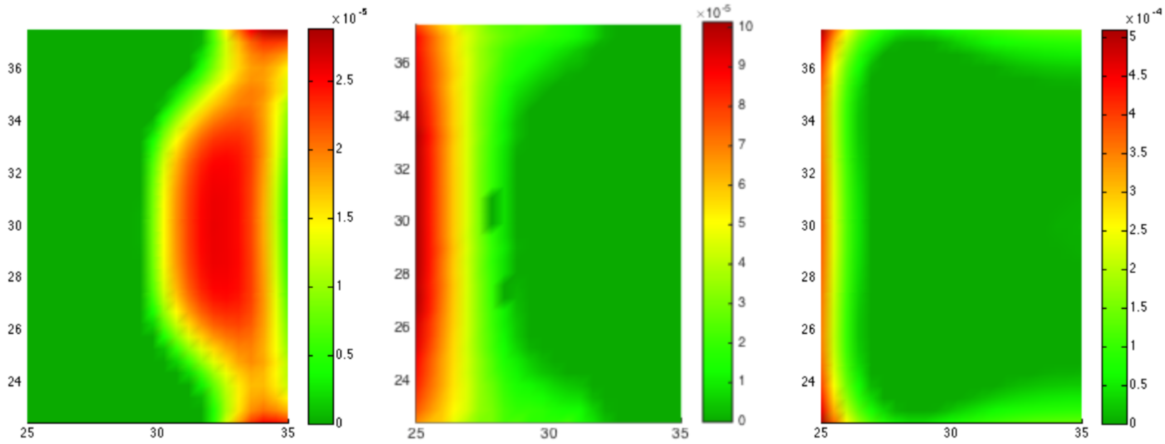


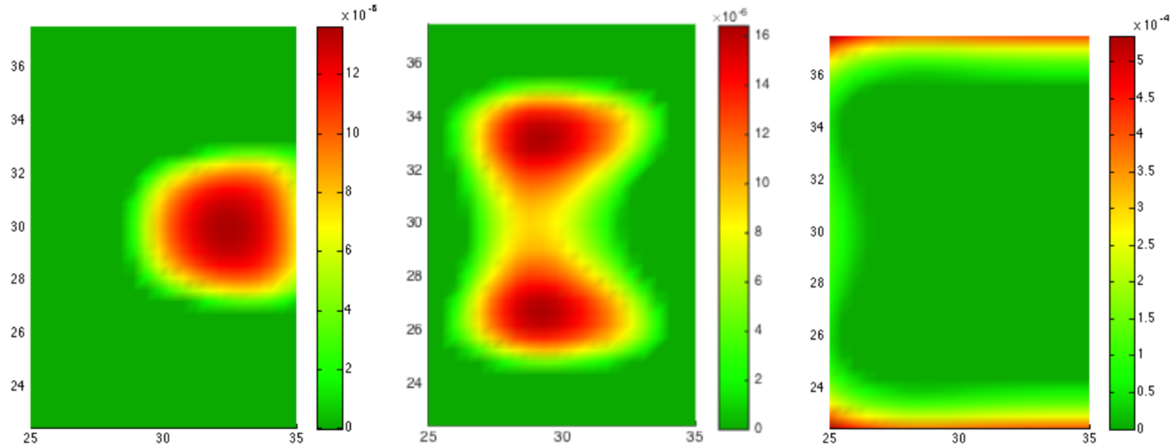
Figure 9: The figures show the displacements of the concrete slab, in a) Full FEA model, b) Variable modulus of subgrade reaction, c) Uniform modulus of subgrade reaction.

To investigate how the model could affect the design of a rather simple raft foundation we have chosen to calculate the reinforcement needed in the slab, for the case of a thickness of 250 mm and a value of $q_1 = 50$ kN/m using the method described in [6]. The bending and twisting moments of the slab are needed to evaluate the required reinforcements. The bending moments are computed as the second derivative of the vertical displacements computed in the finite element analysis, multiplied by the plate bending stiffness [6]. The in-plane normal forces have been evaluated from the integral of the stress with respect to the thickness of the concrete slab. Using the advanced three-layer model it is possible to determine the minimum required reinforcements in the top and bottom of the concrete slab. Figures 10-11 show the required reinforcement in the bottom of the foundation. Studying the results in Figure 10, it can be seen that the reinforcement requirements in the x-direction are in poor agreement also when comparing the full FE-model to the model using a variable value of the modulus of subgrade reaction. The reinforcement needs in the y-direction are presented in Figure 11. Figures 12-13 show the required reinforcement in the top of the of the concrete slab. It should be noticed that results in Figure 13c indicated that reinforcements are needed close to the centre of the slab, whereas, Figure 13a and Figure 13b indicates the need for reinforcement along the edge of the slab.



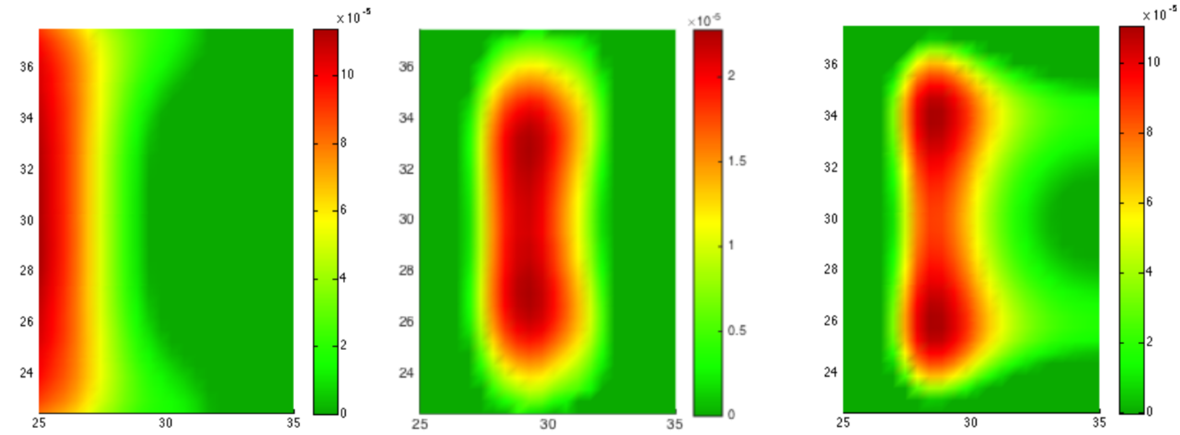
a) b) c)

Figure 10: The calculated reinforcement requirement in the bottom layer of the concrete foundation. The figures show the required amount of reinforcement per meter, m^2/m, in the x-direction. a) Full FEA model, b) Variable modulus of subgrade reaction, c) Uniform modulus of subgrade reaction.



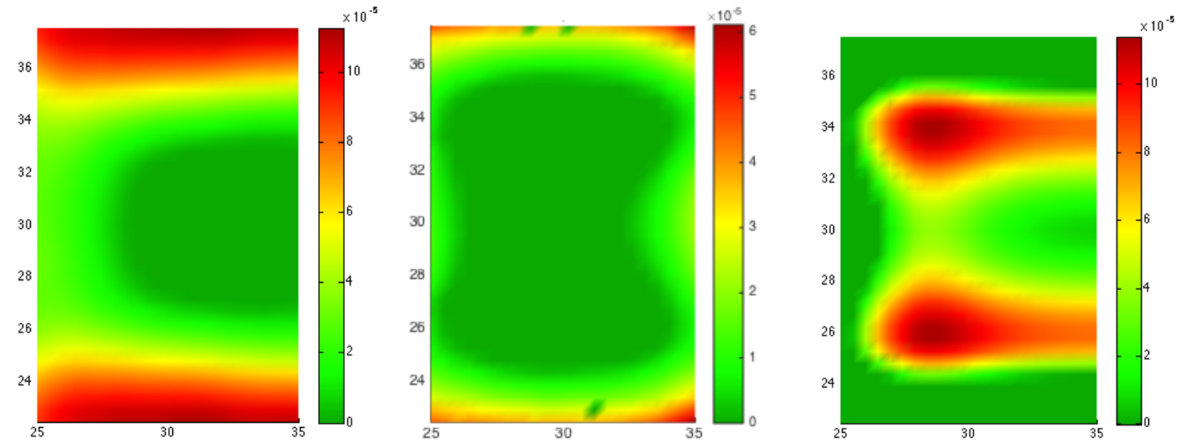
a) b) c)

Figure 11: The calculated reinforcement requirement in the bottom layer of the concrete foundation. The figures show the required amount of reinforcement per meter, m^2/m, in the y-direction. a) Full FEA model, b) Variable modulus of subgrade reaction, c) Uniform modulus of subgrade reaction.



a) b) c)

Figure 12: The calculated reinforcement requirement in the top layer of the concrete foundation. The figures show the required amount of reinforcement per meter, m^2/m, in the x-direction. a) Full FEA model, b) Variable modulus of subgrade reaction, c) Uniform modulus of subgrade reaction.



a) b) c)

Figure 13: The calculated reinforcement requirement in the top layer of the concrete foundation. The figures show the required amount of reinforcement per meter, m^2/m, in the y-direction. a) Full FEA model, b) Variable modulus of subgrade reaction, c) Uniform modulus of subgrade reaction.

4 Concluding remarks

In this paper we have compared the use of subgrade modulus with full finite element analysis in the design of shallow foundations. We have implemented the advanced three-layer model, to compute the reinforcement needs in concrete foundations. From the results it becomes evident that it is important to have a good understanding of the concept behind the modulus of subgrade reactions, if it is to be used to design the reinforcements in concrete slabs. Using an incorrect value of k_s can lead to erroneous design of the reinforcement.

5 Acknowledgements

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